The Sixth “European Students Meeting”
ESM 2011

Forecasting in ICT

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Business Forecasting

The objective of Business forecasting is to enable reliable business decisions

Application: mid-term and long-term business planning or particular business challenges /opportunities:

- New products / services
- New markets or new conditions on existing market (competition, technology changes, ...)

Result of efficient forecasting should be:

- The most probable value of observed indicator
- The interval in which the value of the observed indicator has a particular probability of being in (confidence interval & confidence level)

Forecasting in ICT

Views: operator, service provider, content provider, vendors (telco systems, PC, CPE, handsets, SIMs, …), customers (KA, LA,…), regulator,…

Starting point: demand forecasting – customers growth dynamics

- Planning of resources: human potentials, equipment, space, sales, marketing, call centers, provisioning, fault-repair, etc.
- Planning of finances: CapEx, OpEx, revenue, EBITDA

Literature:

Fildes, R.: Telecommunications demand forecasting – a review http://www.cc.nctu.edu.tw/~etang/Marketing_Research/TelecommunicationsForecasting.pdf

Telektronikk magazine: Telecommunications Forecasting
http://www.telenor.com/telektronikk/volumes/index.php?page=overview&id1=27&select=all
http://www.telenor.com/no/innovasjon/forskning/publikasjoner/telektronikk/volume/telektronikk-3-4-2008

ITU Telecommunication Development Sector (ITU-D) - Adjusting Forecasting Methods to the Needs of the Telecommunication Sector
http://www.itu.int/ITU-D/finance/work-cost-tariffs/events/expert-dialogues/forecasting/presentations.html

Telecommunications forecasting - http://en.wikipedia.org/wiki/Telecommunications_forecasting

Forecasting Methods - Qualitative Methods

Qualitative methods rely exclusively on the intuition of experts, while the statistical analysis of available data is not taken into account. The most important among them are:

Judgmental method – based on the experience of experts who forecast future conditions. The results of forecasting can also be numerically expressed, but are not an outcome of applying analytical or statistical models.

Delphi method – also based on expert knowledge, but with a detailed procedure of reconciling independent predictions of future state, with consensus as a goal.
Useful WEB tool: http://armstrong.wharton.upenn.edu/delphi2/

Scenario method – based on a set of terms that regulate the predicting of future events. Changing conditions results with several possible outcomes concerning an individual case. Taking it all into account, the experts choose the most probable scenario.

Forecasting Methods - Quantitative Methods

Quantitative methods are based on analytical and statistical models of the observed phenomenon. It is presumed, for the forecasting purposes, that the developed models will also be valid for the phenomenon description in the future.

The most important methods are:

Time series methods – predict the future based on the extrapolation of the available past information.

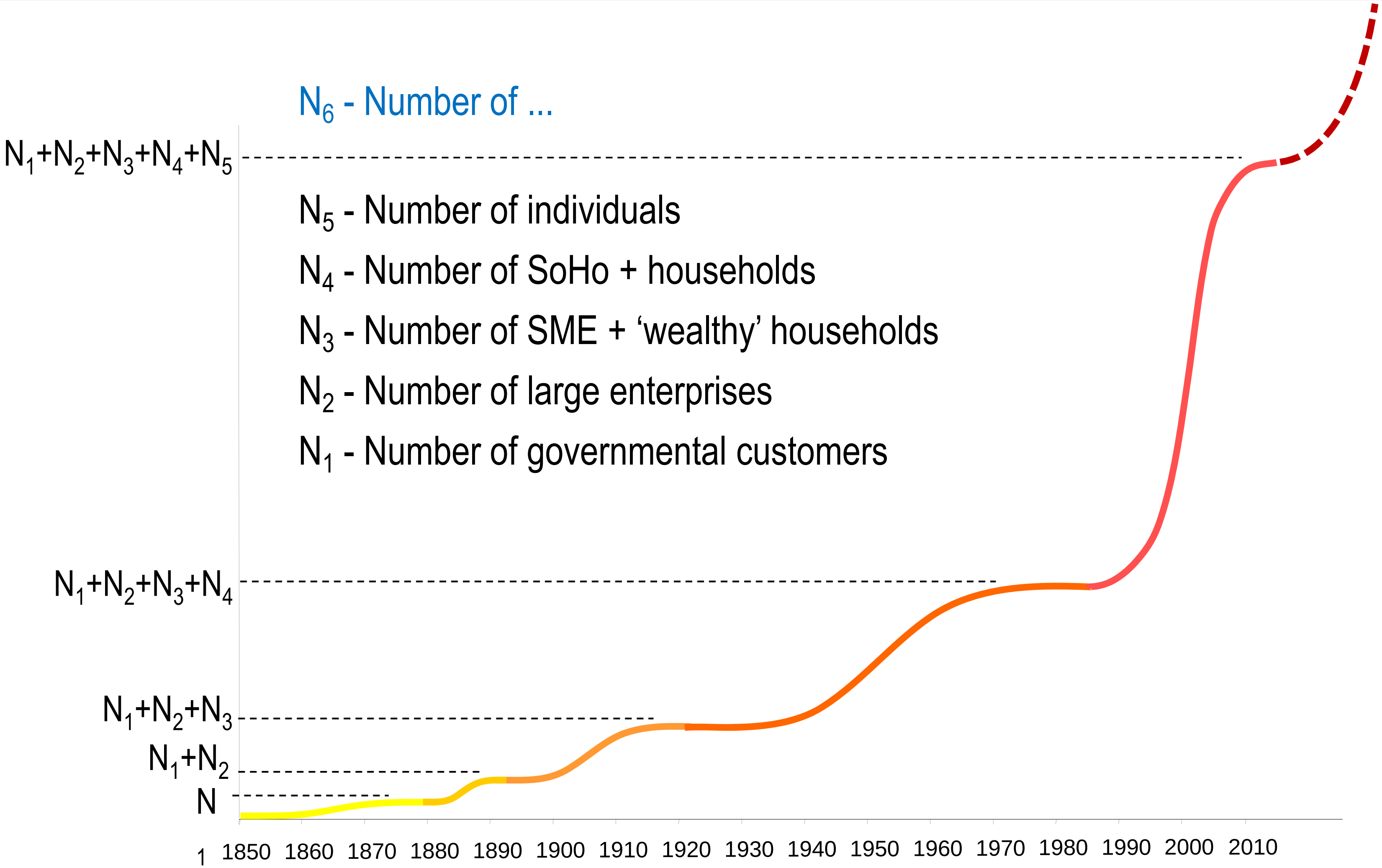
Causal methods – recognize the relations between the variables which are to be forecasted and the independent variables which can be interpreted. Their elements are regression models and various techniques for the evaluation of their applicability, as well as the reliability of forecasting results.

Literature:

Armstrong, J. S. (Eds): Principles of Forecasting: A Handbook for Researchers and Practitioners, Kluwer Academic Publishers, 2001

Mostly available on Forecasting principles portal: http://www.forecastingprinciples.com/

Evolution of number of ICT customers



Diffusion of innovation and new technology, subscription services, market adoption of consumer durables, and allocations of restricted resources have S-shaped (sigmoidal) growth.

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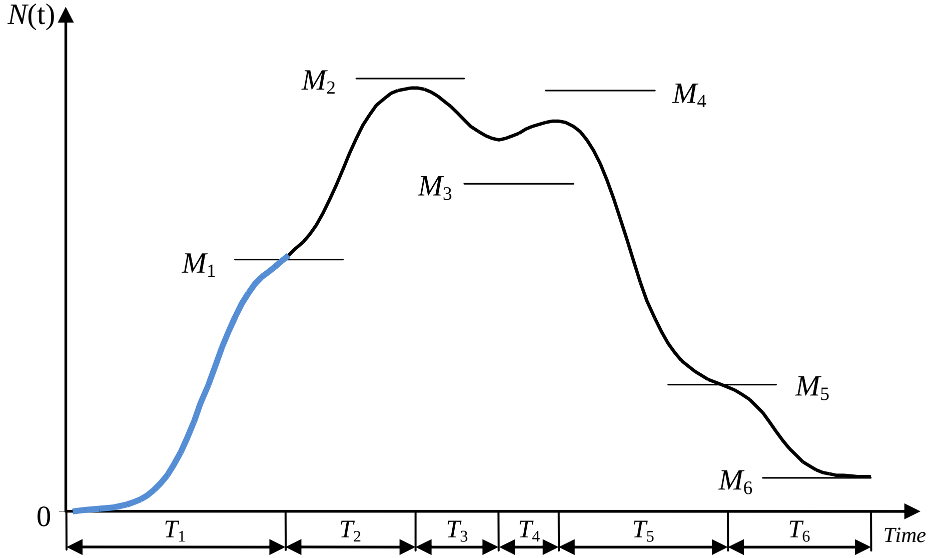
ICT service life-cycle (SLC)



Typical market adoption of service during entire SLC

N(*t*) - number of the users, M_i - market capacities

During the whole service life-cycle (SLC), market capacity changes in hops and resembles a series of stairs.

T_1 Service is unique and new on the market. Its market capacity M_1 is identical to the current total market capacity. Customer growth can me modeled by simple S-curve models.

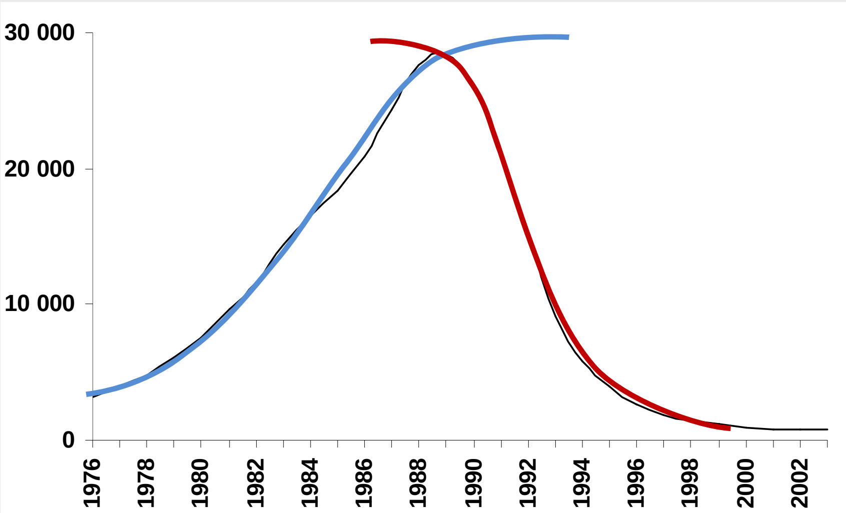
T_2 New market opportunities for that service emerge (economical or technological). Its market capacity and current total market capacity are increased to M_2

T_3 Service is confronted with the first competition in unchanged market capacity. Number of customers *N*(*t*) decreases and service market capacity declines to M_3 level

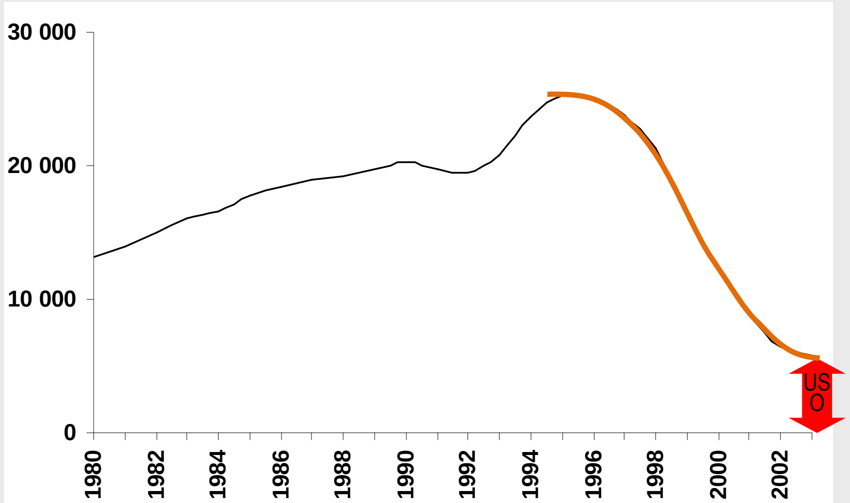
T_4 Counter-attack of observed service provider occurs – certain number of customers are coming back and/or new customers are captured. Service market capacity is increased to M_4.

T_5&T_6 Further attacks from competitive service(s) lead to the number of users *N*(*t*) and market capacity *M* decrease. Competitive service can be identical service but offered by other provider(s), or similar, but technologically more advanced service(s). The last part of SLC is characterized with service obsolesce, substitution by new technology and service disappearance form the market

ICT service life-cycle - Examples



Number of telex subscribers in Portugal 1976-2003



Number of public payphones in Finland 1980-2003

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Growth Models

- Modification of growth models for forecasting purposes
- Determination of optimal model parameters

Literature:

Makridakis, S., S. Wheelwright, R. Hyndman: Forecasting: Methods and Applications (3rd edition), Wiley, 1998

Meade, N. and T. Islam: Modelling and forecasting the diffusion of innovation – A 25-year review, International Journal of Forecasting, Vol 22, No. 3 (2006), pp 519-545

Growth models – Modification for forecasting purposes

Modifications:

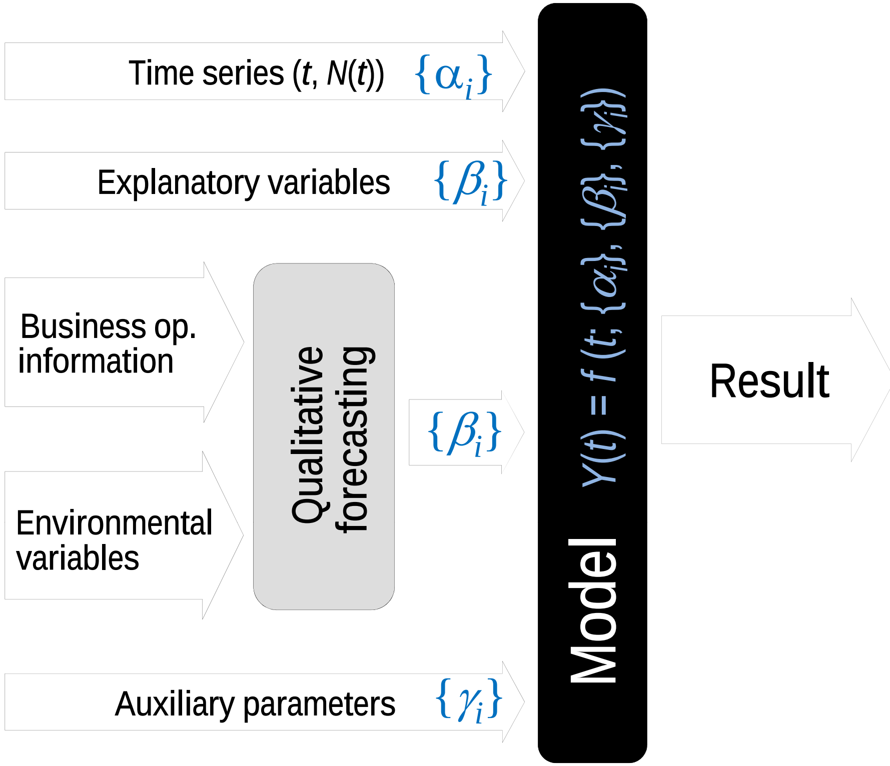
→ to accept external variables as model parameters:
- explanatory marketing variables,
- business operations information,
- environmental variables;

→ Growth/decline of each segment of service life-cycle is S shaped.

$\{\alpha_i\}$ Set of model parameters resulting from fit of time series history

$\{\beta_i\}$ Set of explanatory parameters - resulting from qualitative forecasting; e.g. t_s – time of launch; t_e, $N(t_e)$ – target point in the future; M – (local) market capacity of service; t_m – time of peak of sales, etc

$\{\gamma_i\}$ Set of auxiliary parameters which allows forecasting practitioner to adapt model to her/his specific needs.



Growth models – Modification for forecasting purposes

Environmental variables (BI – business intelligence):

- Customers
- Competition
- Influence of similar services
- Technology
- Macroeconomics
- Regulation

Business operations information (internal knowledge):

- Strategy
- Present and planned resources (financial, HW/SW systems, HR, space, ...)
- Planned date of service launch / service cancellation
- Service provision and activation ability
- Ability of sales and marketing
- Ability of vendors and partners
- IT - CRM / DWH
- ...

Growth models – Determination of optimal parameters

Number of customers modeling:

$$y(t) = f(t; a_1, a_2, \ldots, a_k)$$

k free parameters – at least *k* known data points: $(t_i, N(t_i))$

Case: Exactly $n = k$ data points are available

System of equations : $N(t_i) - f(t_i; a_1, a_2, \ldots, a_k) = 0, \quad i = 1, \ldots, k$

Case: Available n, $n > k$ data points are available:

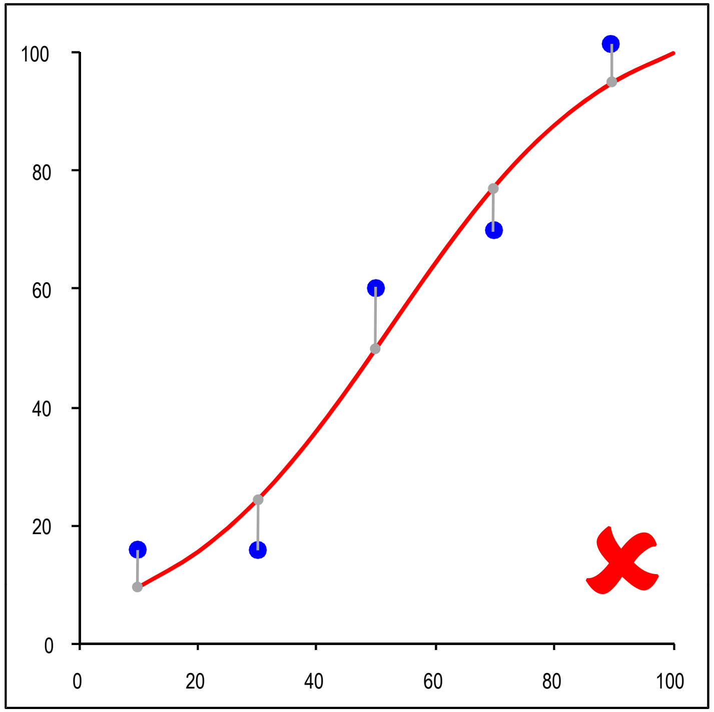
⇨ Weighted least squares method

Objective is to minimize sum of squared difference between data points and model evaluated points:

$$S = \sum_{i=1}^{n} w_i \cdot \left[f(t_i; a_1, a_2, \ldots, a_k) - N(t_i) \right]^2$$

Growth models – Determination of optimal parameters

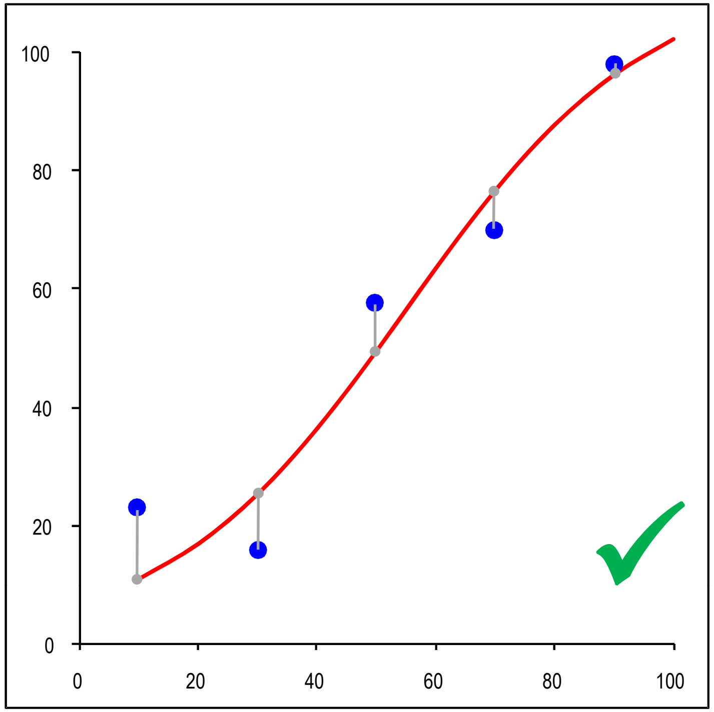
Ordinary least squares method (OLS)



$$\min_{\{a_1 \dots a_k\}} \sum_{i=1}^{n} \left[f(t_i; a_1, \dots, a_k) - N(t_i) \right]^2$$

Values obtained for parameters are statistically smoothed, i.e. the influence of particular measurement errors of *N*(*t*) is reduced

Weighted least squares method

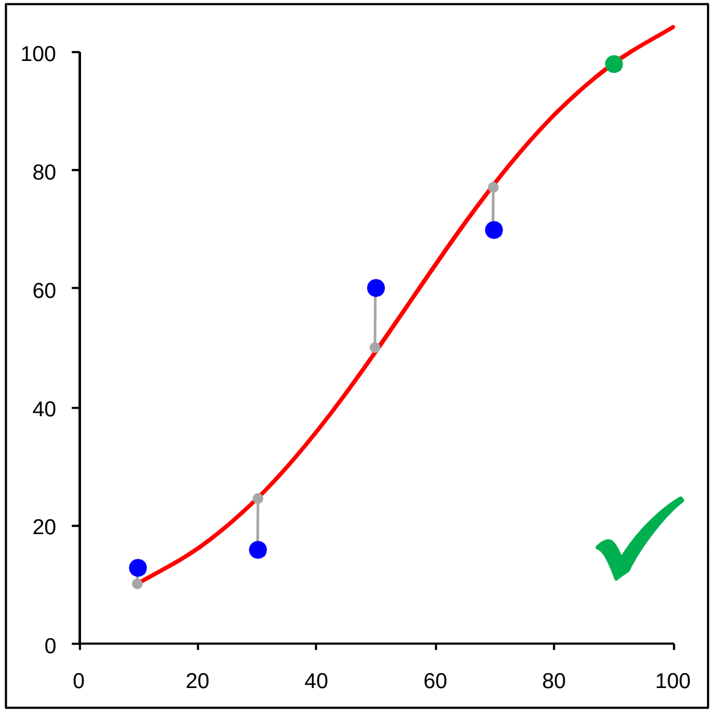


$$\min_{\{a_1 \dots a_k\}} \sum_{i=1}^{n} w_i \cdot \left[f(t_i; a_1, \dots, a_k) - N(t_i) \right]^2$$

Introduction of weights w_i ⇨ focus can be set on the time interval near the last observed data

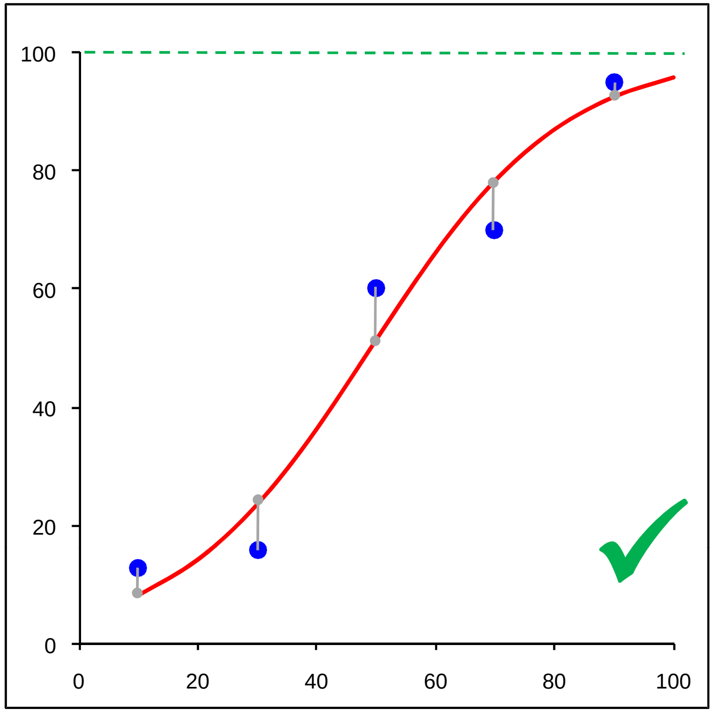
Growth models – Determination of optimal parameters

Ordinary least squares method with fixed value of the last data point $(t_f, N(t_f))$



$$\min_{\{a_1 \ldots a_{k-1}\}} \sum_{i=1, i \neq f}^{n} \left[f(t_i; a_1, \ldots, a_{k-1}; t_f, N(t_f)) - N(t_i) \right]^2$$

Ordinary least squares method with fixed value of a parameter a_k



$$\min_{\{a_1 \ldots a_{k-1}\}} \sum_{i=1}^{n} \left[f(t_i; a_1, \ldots, a_k) - N(t_i) \right]^2$$

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Models for the First Segment of SLC

- The logistic model
- The Bass model

Literature:

http://www.telenor.com/no/resources/images/144-154_GrowthModels-ver1_tcm26-36191.pdf

The logistic model

Describes growth of the number customers observed over time in a closed market, without the impact of any other service

Differential form:

$$\frac{dL(t)}{dt} = aL(t)\cdot\left(1-\frac{L(t)}{M}\right)$$

Exponential growth — *Negative feedback*

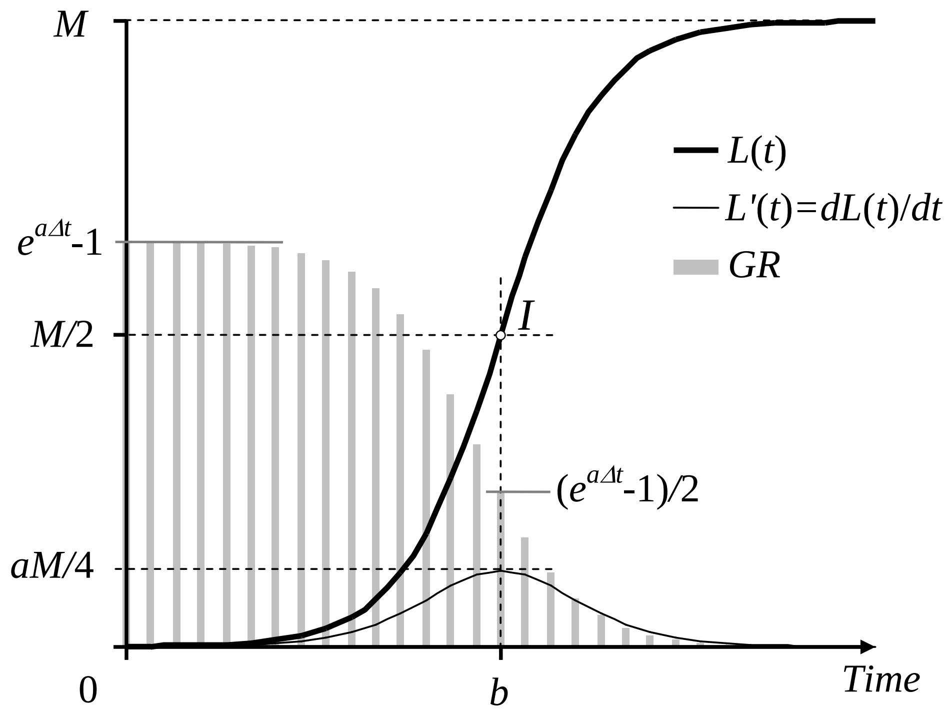
Analytical form:

$$L(t;M,a,b) = L(t) = \frac{M}{1+e^{-a(t-b)}}$$

M - market capacity

a - growth parameter (for a<0 decline)

b - time shift parameter



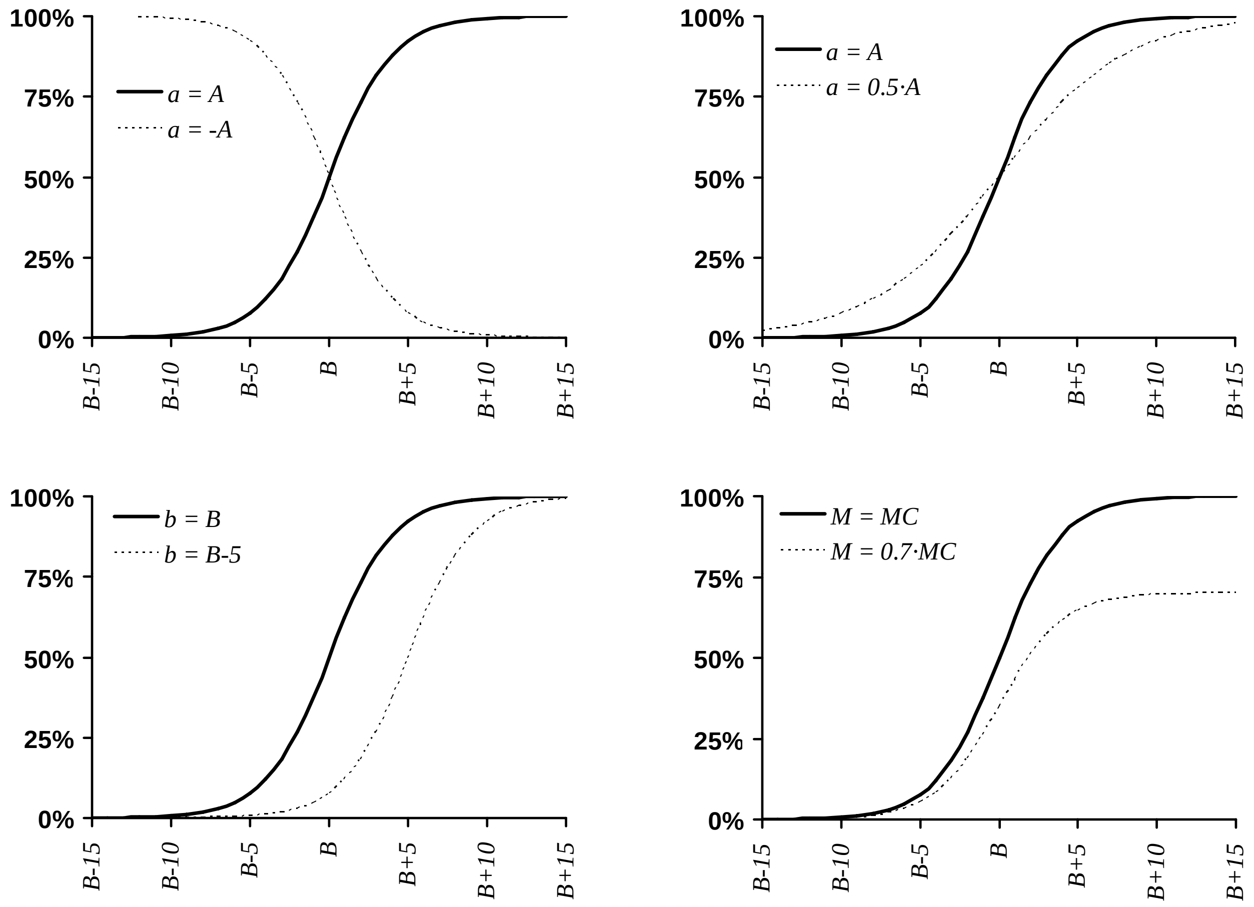
⇨ Inflexion for *t* =*b*, when L(b) = *M* / 2 (maximum of sales)

⇨ S-curve is centro-symmetric regarding inflexion point I (*b*, M/2):

⇨ "Hardly starts to grow up" problem. i.e. *t* for which L(*t*) = 0 does not exist!

The logistic model

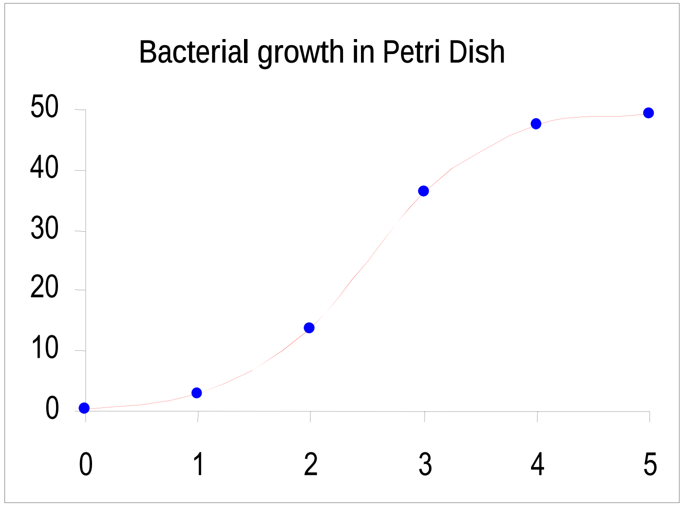
Effect of logistic model parameter change on the form of S-curve

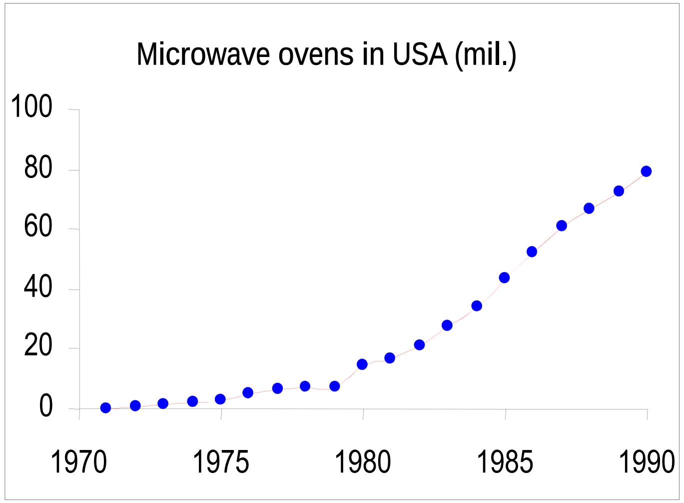


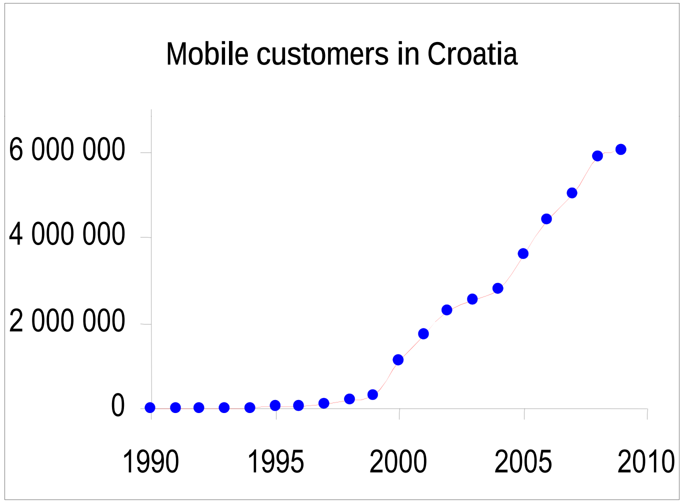
The logistic model - examples

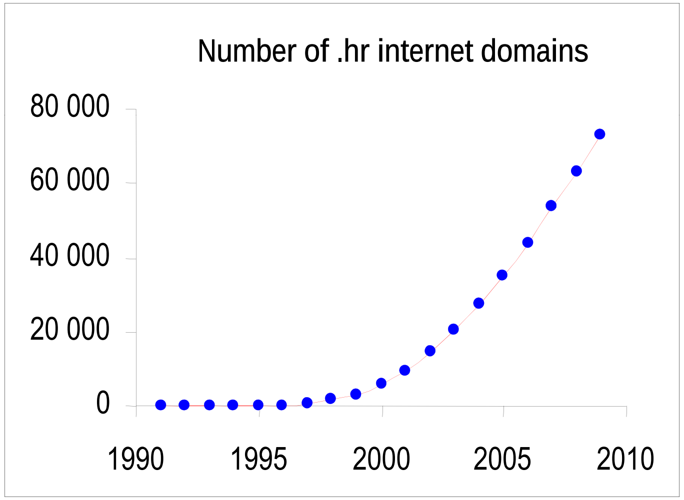
Applications:

- Biological growth
- Adoption of consumer durables
- Subscription services
- Diffusion of innovation and new technology
- Allocations of restricted resources









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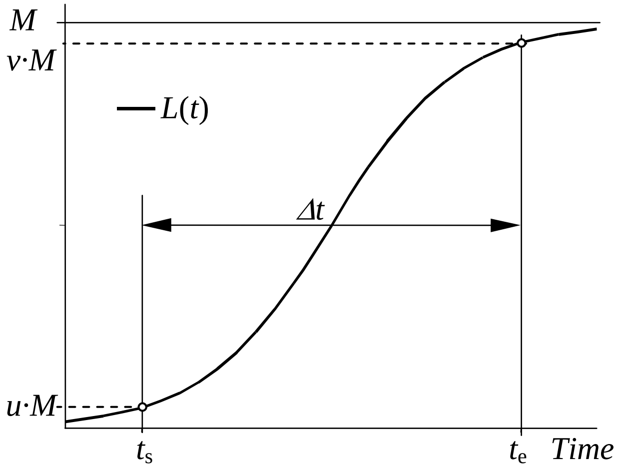
The logistic model through two fixed points

Embedded values of two (known) data points: (t_s , $u{\cdot}M$) and (t_e , $v{\cdot}M$) :



$$a = \frac{1}{\Delta t}\left[\ln\left(\frac{1}{u}-1\right)-\ln\left(\frac{1}{v}-1\right)\right]$$

$$b = t_s + \Delta t \frac{\ln\left(\frac{1}{u}-1\right)}{\ln\left(\frac{1}{u}-1\right)-\ln\left(\frac{1}{v}-1\right)}$$

Condition: $0 < u < v < 1$; Δt = time to saturation

Case of symmetrical values for u and $v = 1 - u$:

$$a = \frac{2}{\Delta t}\ln\left(\frac{1}{u}-1\right) \qquad b = t_s + \frac{\Delta t}{2}$$

⇨ model has form:

$$L(t; M, t_s, \Delta t, u) = \frac{M}{1+\left(\frac{1}{u}-1\right)^{1-2(t-t_s)/\Delta t}}$$

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The logistic model through two fixed points

Framework for forecasting of new services adoption prior to launch:

	$u = 5$ %, $v = 95$ %	$u = 10$ %, $v = 90$ %
$\Delta t = 2$ years	$N(t) = \dfrac{M}{1 + e^{-2.944(t - t_s - 1)}}$	$N(t) = \dfrac{M}{1 + e^{-2.197(t - t_s - 1)}}$
$\Delta t = 5$ years	$N(t) = \dfrac{M}{1 + e^{-1.178(t - t_s - 2.5)}}$	$N(t) = \dfrac{M}{1 + e^{-0.879(t - t_s - 2.5)}}$
$\Delta t = 10$ years	$N(t) = \dfrac{M}{1 + e^{-0.589(t - t_s - 5)}}$	$N(t) = \dfrac{M}{1 + e^{-0.439(t - t_s - 5)}}$
$\Delta t = 15$ years	$N(t) = \dfrac{M}{1 + e^{-0.393(t - t_s - 7.5)}}$	$N(t) = \dfrac{M}{1 + e^{-0.293(t - t_s - 7.5)}}$

According to: T. Modis - *Conquering Uncertainty*, McGraw-Hill, 1998:

- **Services** consist of units sold that have typical life-cycle of 6 to 10 quarters
- **Service families** consist of related services that have a typical business cycle of 5 years
- **Basic technologies** consist of a set of related service families that have a typical cycle of 10 to 15 years

The Bass model

Introduces the effect of innovators via coefficient of innovation *p*, which corrects deficiency of simple logistic growth

Differential form:

$$\frac{dB(t)}{dt} = \underbrace{qB(t)\left(1-\frac{B(t)}{M}\right)}_{\text{Effect of imitators (Logistic growth)}} + \underbrace{p\left(M - B(t)\right)}_{\text{Effect of innovators}}$$

Analytical form:

$$B(t;M,p,q,t_s) = B(t) = M\,\frac{1-e^{-(p+q)(t-t_s)}}{1+\frac{q}{p}e^{-(p+q)(t-t_s)}}$$

M - market capacity

p - coefficient of innovation, $p > 0$

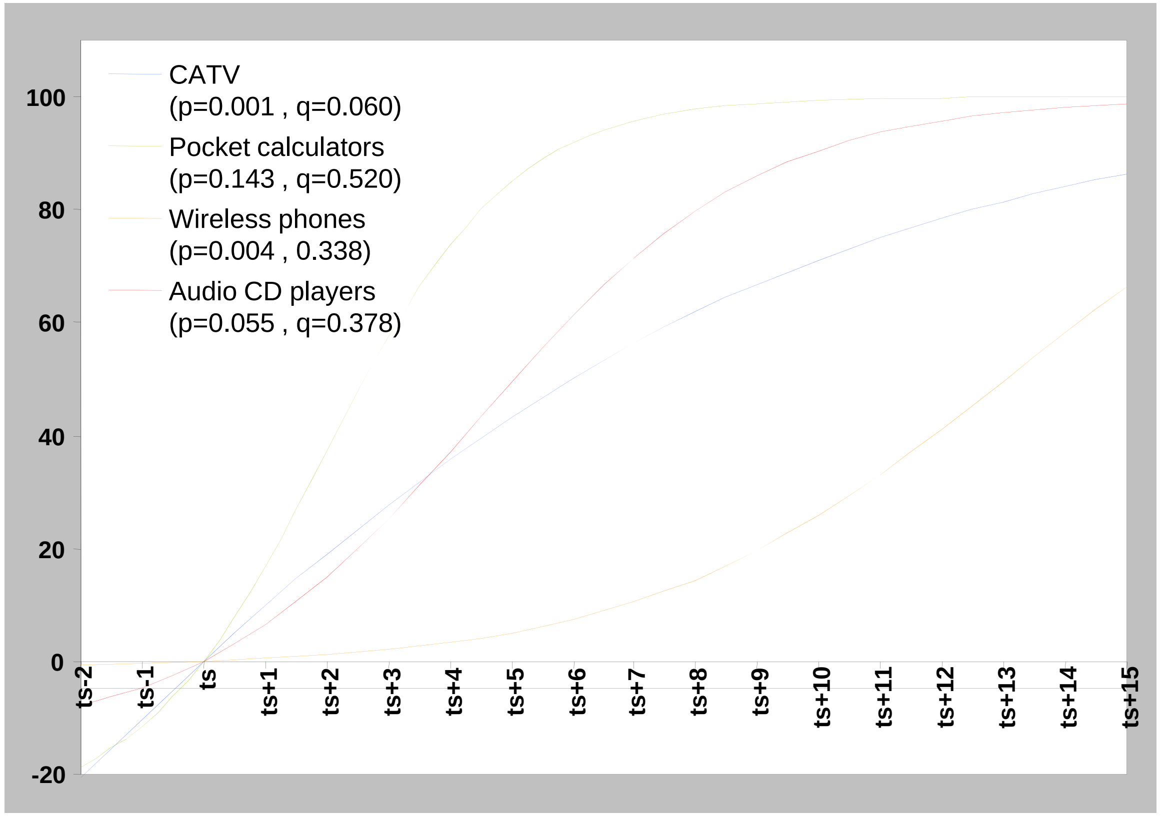
q - coefficient of imitation, $q \geq 0$

t_s - time when service is introduced, $B(t_s)=0$

⇨ 4 free parameters

⇨ shape of S-curve similar to the logistic growth model, but shifted down on *y*-axis

The Bass model - Examples of durables diffusion



For all product t_s is fixed and M is set to 100.

Literature: Bass Basement Research Institute: http://bassbasement.org
Data: Predicting the speed of technology introduction http://andorraweb.com/bass

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The Bass model - Examples of durables diffusion

Databases and software tools (e.g. *GBASS*):

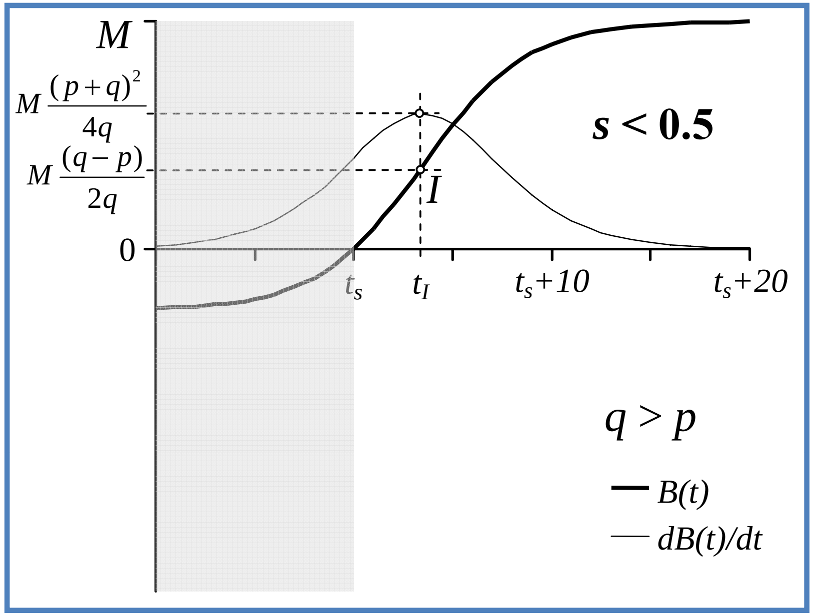
Product/Technology	Period of Analysis	*p*	*q*	*m*
Electrical Appliances				
Room air conditioner	1950-1979	.006	.185	60.5
Bed cover	1949-1979	.008	.130	72.2
Blender	1949-1979	.000	.260	54.5
Can opener	1961-1979	.050	.126	68.0
Electric coffee maker	1955-1979	.042	.103	100.0
Clothes dryer	1950-1979	.009	.143	70.1
Clothes washer	1923-1971	.016	.049	100.0
Coffee maker ADC	1974-1979	.077	1.106	32.2
Curling iron	1974-1979	.101	.762	29.9
Dishwasher	1949-1979	.000	.213	47.7
Disposer	1950-1979	.000	.179	50.4
Fondue	1972-1979	.166	.440	4.6
Freezer	1949-1979	.019	.000	94.2
Frypan	1957-1979	.142	.000	65.6
Hair dryer	1972-1979	.055	.399	51.6
Hot plates	1932-1979	.056	.000	26.3
Microwave oven	1972-1990	.002	.357	91.6
Mixer	1949-1979	.000	.134	97.7
Power leaf blower (gas or electric)	1986-1996	.013	.315	26.0
Range	1925-1979	.004	.065	63.6
Range, built-in	1957-1979	.048	.086	21.7
Refrigerator	1926-1979	.025	.126	99.7
Slow cooker	1974-1979	.000	1.152	34.4
Steam iron	1950-1979	.031	.128	100.0
Toaster	1923-1979	.038	.000	100.0
Consumer Electronics				
Cable television	1981-1994	.100	.060	68.0
Calculators	1973-1979	.143	.520	100.0
Camcorder	1986-1996	.044	.304	30.5
CD player	1986-1996	.055	.378	29.6
Cellular telephone	1986-1996	.008	.421	45.1
Cordless telephone	1984-1996	.004	.338	67.6
Electric toothbrush	1991-1996	.110	.548	14.8
Home PC (millions of units)	1982-1988	.121	.281	25.8
Radio	1922-1934	.027	.435	100.0
Telephone answering device	1984-1996	.025	.406	69.6
Television, black and white	1949-1979	.108	.231	96.9
Television, color	1965-1979	.059	.146	100.0
VCR	1981-1994	.025	.603	76.3

Lilien, G. L., A. Rangaswamy, C. Van den Bulte, Diffusion Models: Managerial Applications and Software, New-Product Diffusion Models pp. 295-336, Kluwer Academic Publishers

The Bass model

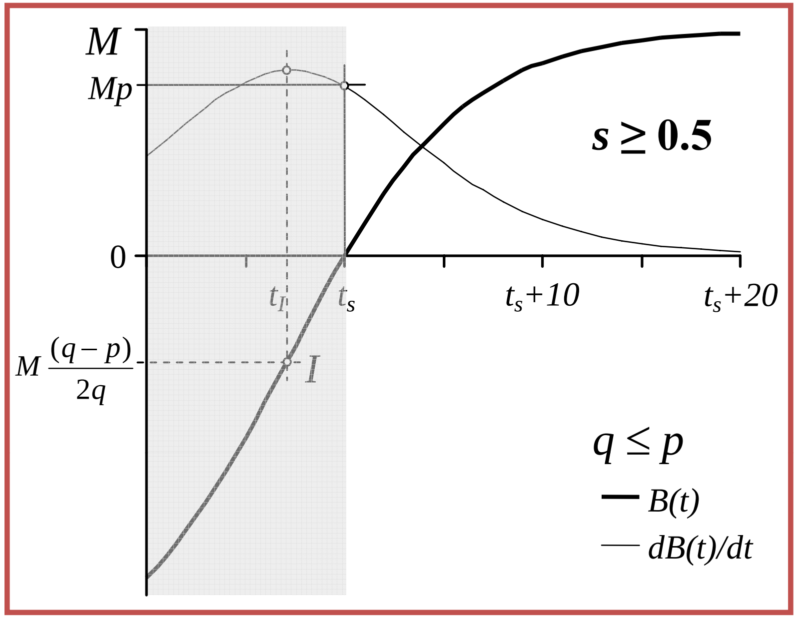
Characteristic values and points of the Bass model of growth

Inflexion point is after service launch



Imitators prevail

Inflexion point is before service launch



Innovators prevail

Near inflexion point t_{I} sales is maximal!

Excel example

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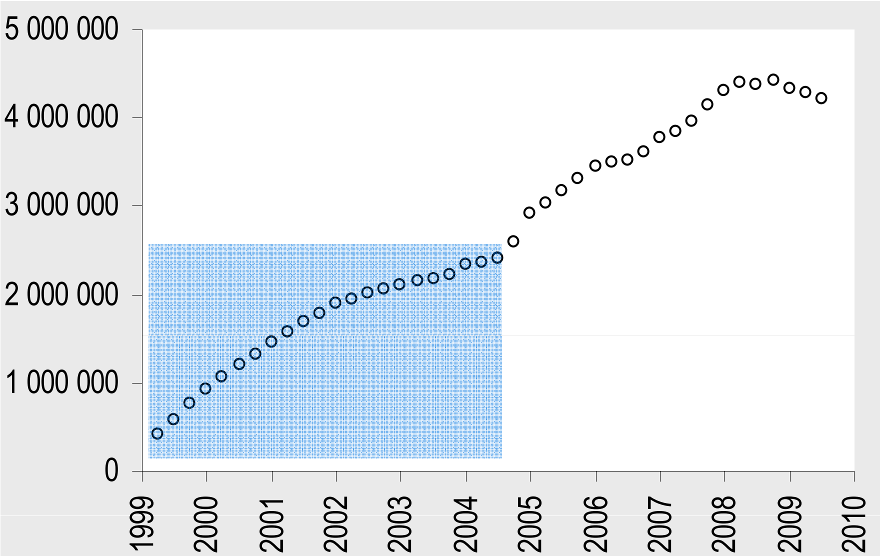
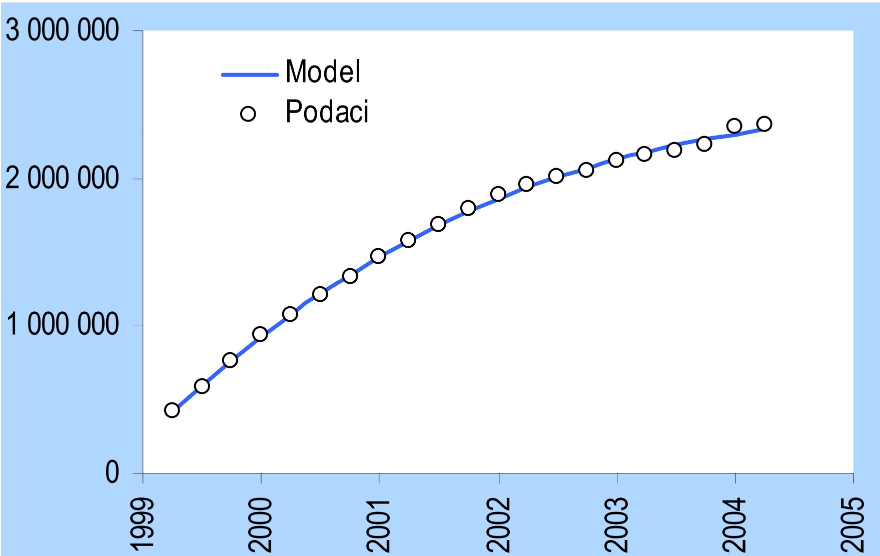
Example: Prepaid customers of cellular mobile networks in Croatia





Quarter	Decimal time	Data	Model
Q1 2000	1999.25	425 078	416 114
Q2 2000	1999.50	585 520	595 057
Q3 2000	1999.75	765 009	764 857
Q4 2000	2000.00	932 490	925 047
Q1 2001	2000.25	1 069 899	1 075 345
Q2 2001	2000.50	1 205 473	1 215 638
Q3 2001	2000.75	1 330 777	1 345 965
Q4 2001	2001.00	1 469 382	1 466 494
Q1 2002	2001.25	1 580 179	1 577 502
Q2 2002	2001.50	1 681 662	1 679 355
Q3 2002	2001.75	1 787 853	1 772 481
Q4 2002	2002.00	1 890 128	1 857 356
Q1 2003	2002.25	1 950 434	1 934 488
Q2 2003	2002.50	2 005 313	2 004 397
Q3 2003	2002.75	2 052 803	2 067 608
Q4 2003	2003.00	2 111 900	2 124 640
Q1 2004	2003.25	2 154 800	2 175 995
Q2 2004	2003.50	2 181 950	2 222 158
Q3 2004	2003.75	2 232 100	2 263 587
Q4 2004	2004.00	2 348 900	2 300 716
Q1 2005	2004.25	2 357 100	2 333 948

The Bass model - results:

M =	2 603 238	v =	95%
p =	0.30676	Δt=	7.08
q =	0.17825	t_I	1997.59
t_S =	1998.71	$(t_I - t_S)/\Delta t$ =	-15.8%

Source: WirelessIntelligence on-line business intelligence database
https://www.wirelessintelligence.com/index.aspx

The Bass model with explanatory parameters

Framework for forecasting of new services adoption prior to launch (assumed: Δt and t_I)

	$(t_I-t_s)/\Delta t$ =	-20%	-10%	0%	10%	20%	30%	40%	50%	60%	70%
v = 95%	p =	2.2480/Δt	2.0585/Δt	1.8318/Δt	1.5654/Δt	1.2605/Δt	0.9257/Δt	0.5850/Δt	0.2853/Δt	0.0866/Δt	0.0102/Δt
	q =	1.1413/Δt	1.4494/Δt	1.8318/Δt	2.3054/Δt	2.8921/Δt	3.6211/Δt	4.5346/Δt	5.7062/Δt	7.3055/Δt	9.8083/Δt
v = 90%	p =	1.7231/Δt	1.6079/Δt	1.4722/Δt	1.3129/Δt	1.1269/Δt	0.9125/Δt	0.6720/Δt	0.4187/Δt	0.1889/Δt	0.0427/Δt
	q =	0.9996/Δt	1.2127/Δt	1.4722/Δt	1.7906/Δt	2.1858/Δt	2.6842/Δt	3.3275/Δt	4.1865/Δt	5.3995/Δt	7.3030/Δt

Example: Growth dynamics of new service:

M = 1 000 000 market capacity

Δt = 10 years to the service growth saturation

v = 95% ⇨ at the end of 10th year no. of customers is 950 000 (penetration is 95%)

Maximum of sales is assumed at the end of 3rd year form service launch (t_I = 3) ⇨ $(t_I-t_s)/\Delta t$ = 30%

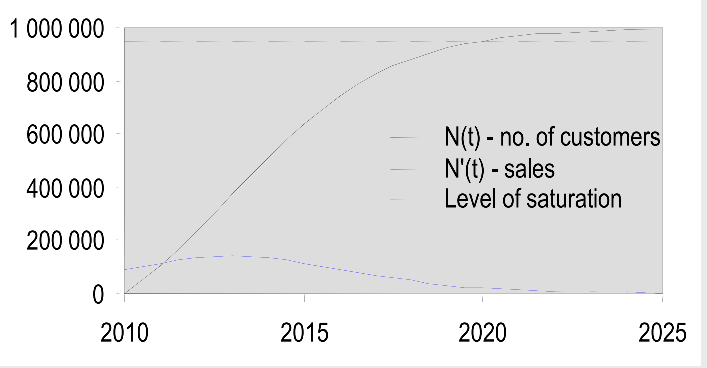
Find p & q from table:

p = 0.9257/10 = 0.09257 — $p + q$ = 0.45468

q = 3.6211/10 = 0.36211 — q / p = 3.91174

t_s = 2010 ⇨ MODEL:

$$N(t) = 1000000 \cdot \frac{1 - e^{-0.45468 \cdot (t-2010)}}{1 + 3.91174 \cdot e^{-0.45468 \cdot (t-2010)}}$$



Models for whole Service Life-Cycle

- Interaction between services
- Multi-Logistic Model

Models for whole Service Life-Cycle

Interaction between services on the market

Only at the beginning of the service life-cycle there is no interaction with other services regarding market adoption, therefore, its growth may be approximated with simple S-shaped growth models (logistic, Bass, Richards)

In latter phases of SLC, interaction between different services is evident, due to:

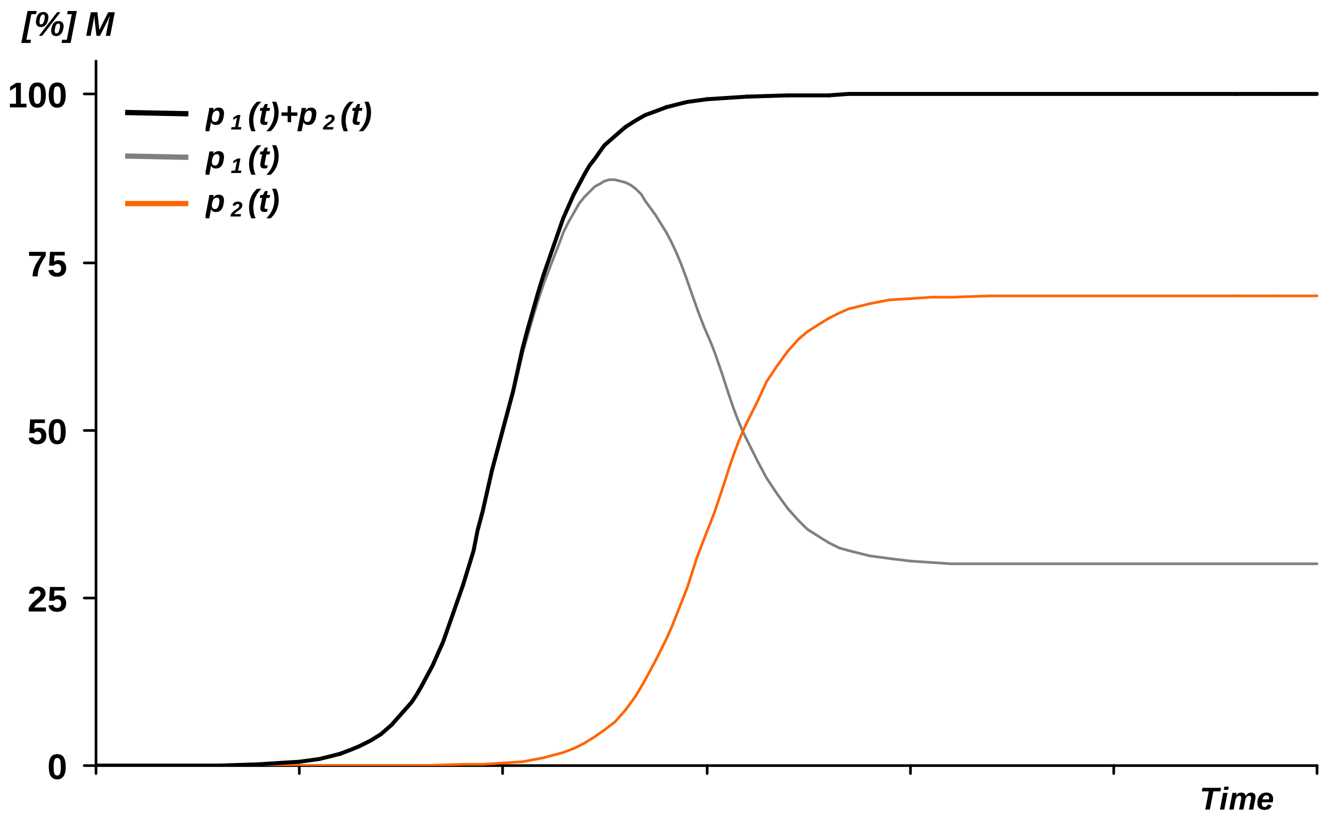
- New market opportunities for service emerge (economical or technological)
- Confrontation with competition: identical service offered by other provider(s), or similar, but technologically more advanced service(s)

Interaction between different services can be divided into three types (combination of types are possible!):

- Service competition
- Service co-evolution
- Service revolution.

Interaction between services - Service competition

Both services are competing in market with unchanged total market capacity:



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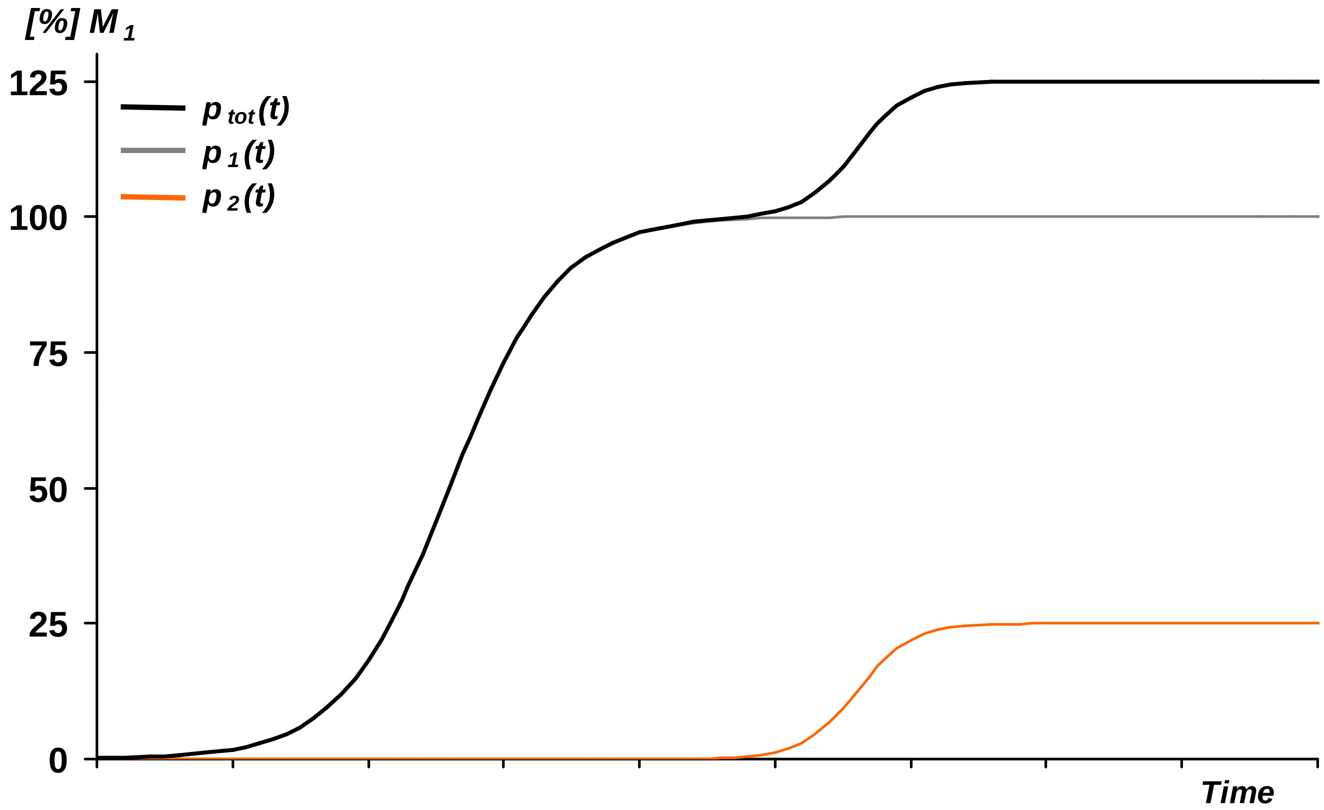
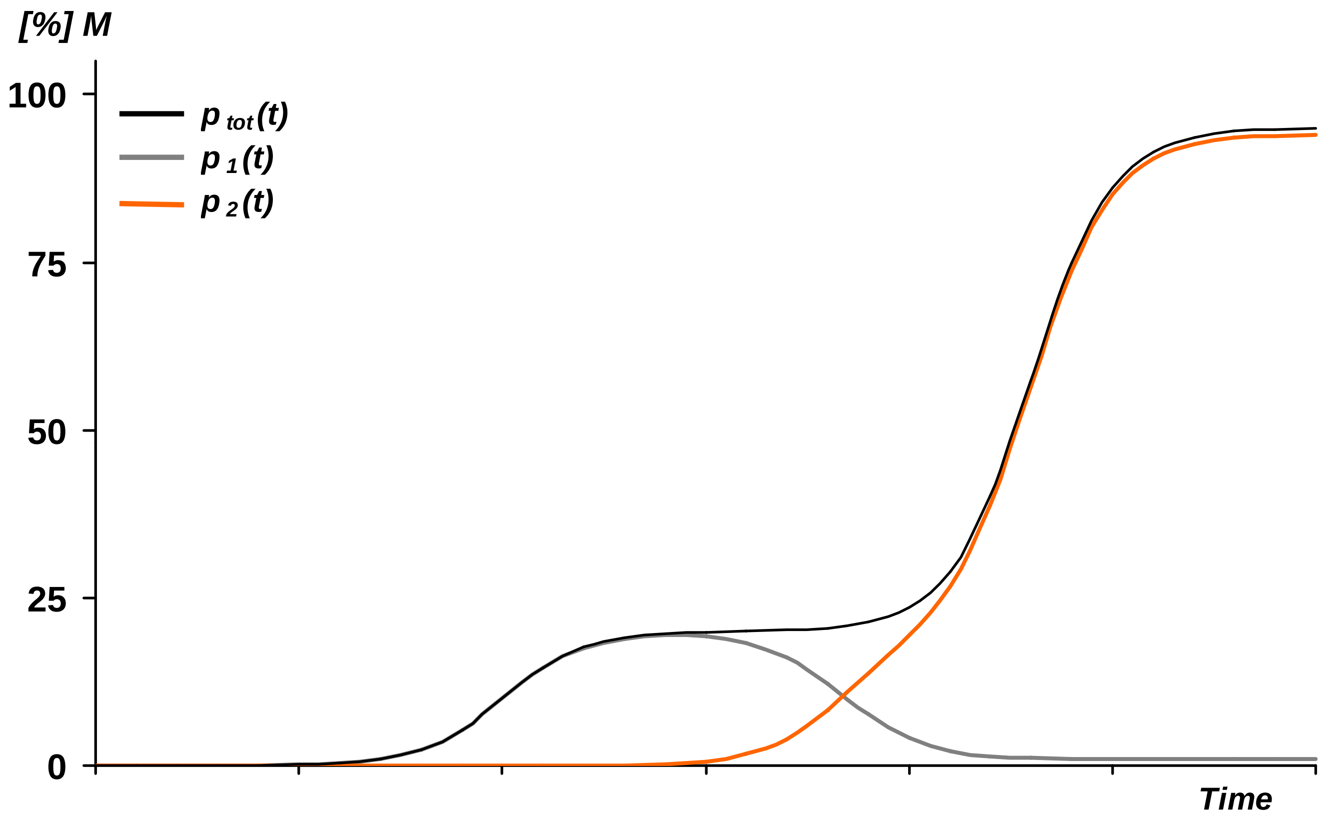
Interaction between services - Service co-evolution

Complementary services change the total market capacity. As a result there is no decrease of existing service penetration:



Interaction between services - Service revolution

New attractive service almost completely eliminates the existing one, total market capacity is noticeably increased:



Forecasting of existing services growth

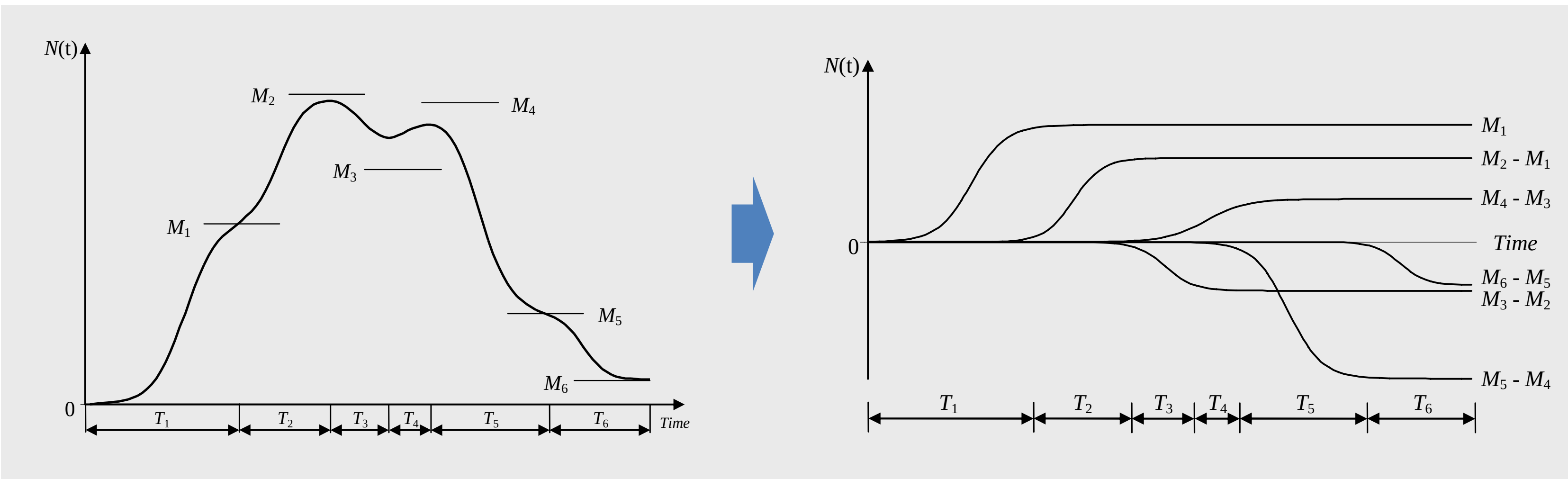
Multi-Logistic Model

$$MLM(t) = M_0 + \frac{M_1 - M_0}{1 + \left(\frac{1}{u} - 1\right)^{1 - 2(t - t_{s1})/\Delta t_1}} + \frac{M_2 - M_1}{1 + \left(\frac{1}{u} - 1\right)^{1 - 2(t - t_{s2})/\Delta t_2}} + \ldots + \frac{M_n - M_{n-1}}{1 + \left(\frac{1}{u} - 1\right)^{1 - 2(t - t_{sn})/\Delta t_n}}$$

Model for the current SLC segment

Model for the first successive SLC segment

<u>Example:</u> Decomposition of a growth dynamics presented on *slide 7* into 6 simple logistic growth model:



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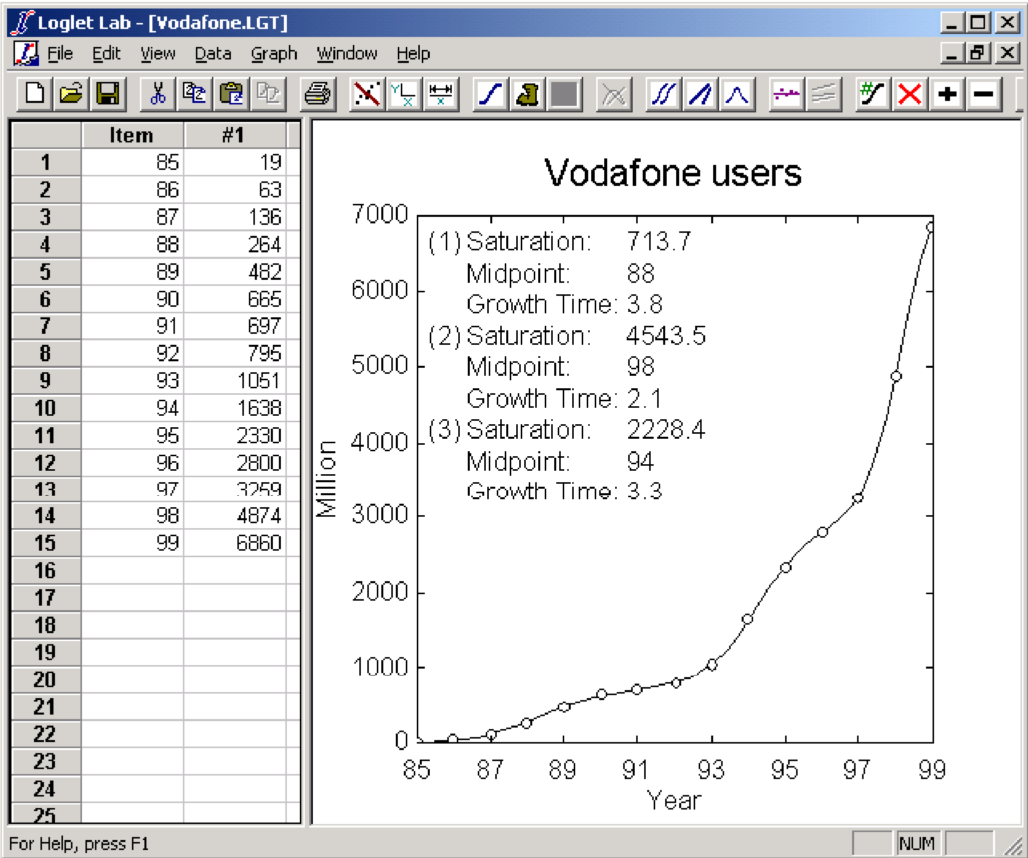
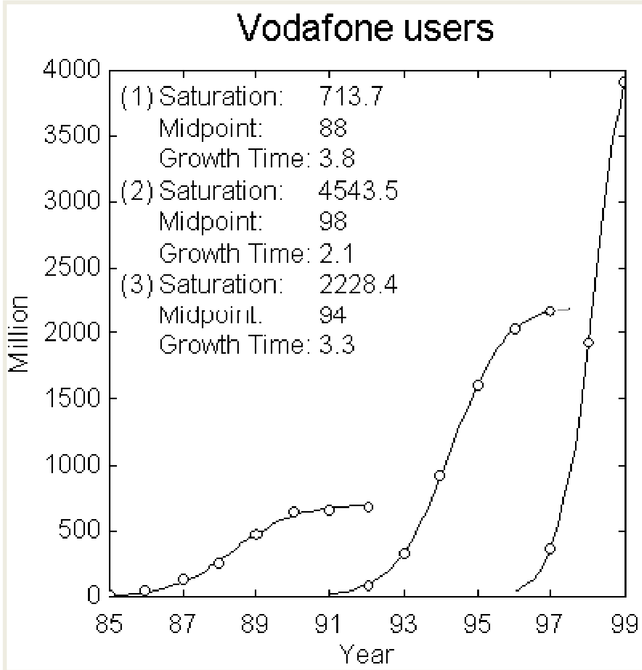
Loglet Lab tool



Decomposition of growth into 3 components:



M = Saturation

b = Midpoint (time shift)

a = 4.3944/ Growth Time

$$a = \frac{2}{\Delta t} ln(\frac{1}{\delta} - 1), \quad \delta = 10\%$$

Loglet Lab by Perrin S. Meyer, Jason Yung and Jesse H. Ausubel

URL: http://phe.rockefeller.edu/LogletLab/

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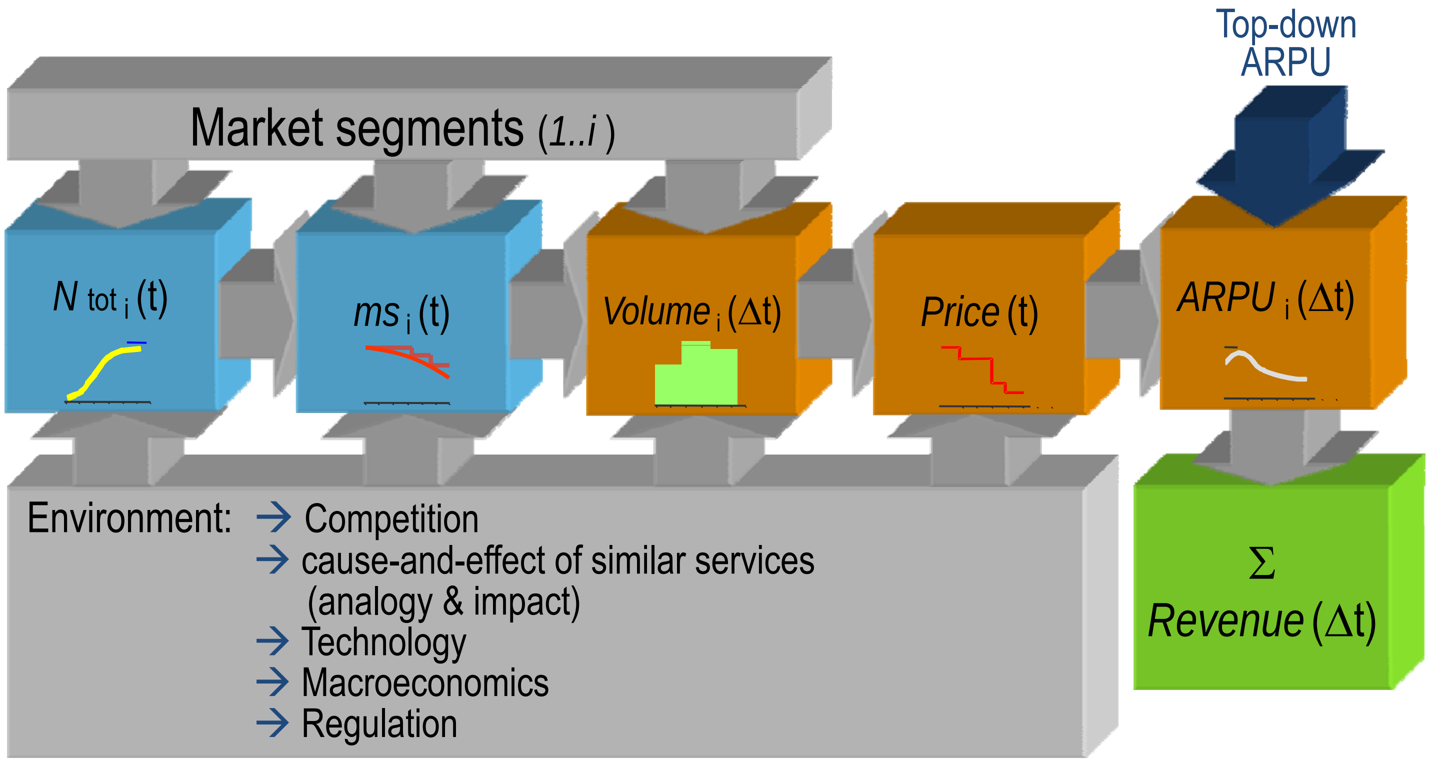
Revenue forecasting

- Bottom-up revenue forecasting flow chart
- Market share modeling and forecasting

Bottom-up Revenue forecasting flow chart



Blocks:

- Growth dynamics forecasting per market segments
- ARPU dynamics forecasting

$N_{tot\,i}$ (*t*) = Number of customers in market segment *i* at time *t*; for all operators on the market (not only for the observed one)

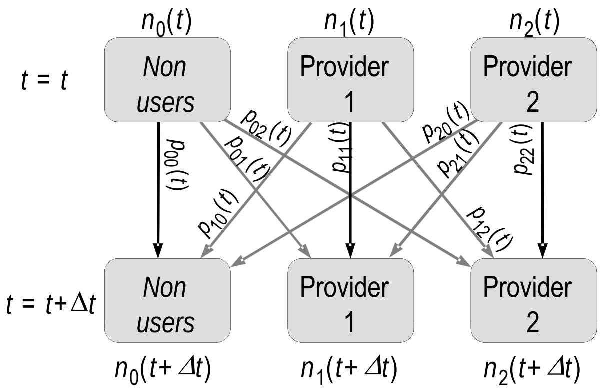
ms_i (*t*) = Market share of chosen operator in market segment *i* at time *t*;

$Volume_i(\Delta t)$ = Standard service usage (traffic) in segment *i* in Δt

Price (*t*) = Price at time *t* of service volume unit

$ARPU_i(\Delta t)$ = average revenue per user/customer in Δt

Market Share Modeling – Markov chains



$$[n_0(t+\Delta t) \quad n_1(t+\Delta t) \quad \cdots \quad n_k(t+\Delta t)] =$$

$$= [n_0(t) \quad n_1(t) \quad \cdots \quad n_k(t)] \times \begin{bmatrix} p_{00}(t) & p_{01}(t) & \cdots & p_{0k}(t) \\ p_{10}(t) & p_{11}(t) & \cdots & p_{1k}(t) \\ \vdots & \vdots & \ddots & \vdots \\ p_{k0}(t) & p_{k1}(t) & \cdots & p_{kk}(t) \end{bmatrix}$$

$$ms_i(t) = \frac{N_i(t)}{\sum_{i=1}^{k} N_i(t)} = \frac{M \cdot n_i(t)}{M \cdot \sum_{i=1}^{k} n_i(t)} = \frac{n_i(t)}{1 - n_0(t)}$$

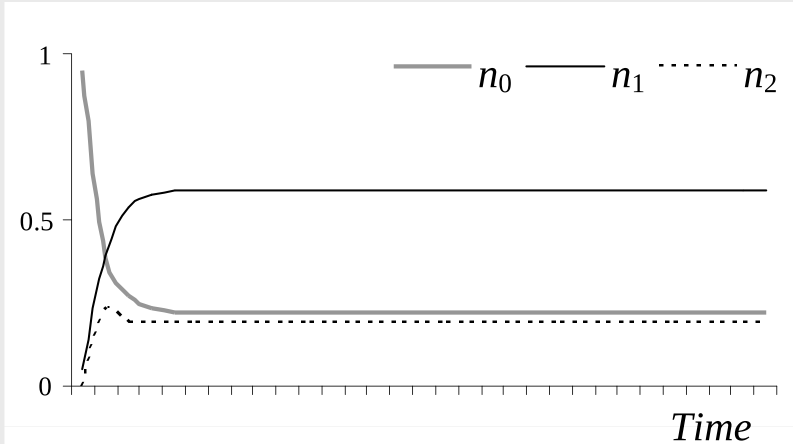
Descriptive features:

$$ChurnRate_i(t) = (1 - p_{ii}), \quad i = 1,\dots,k$$

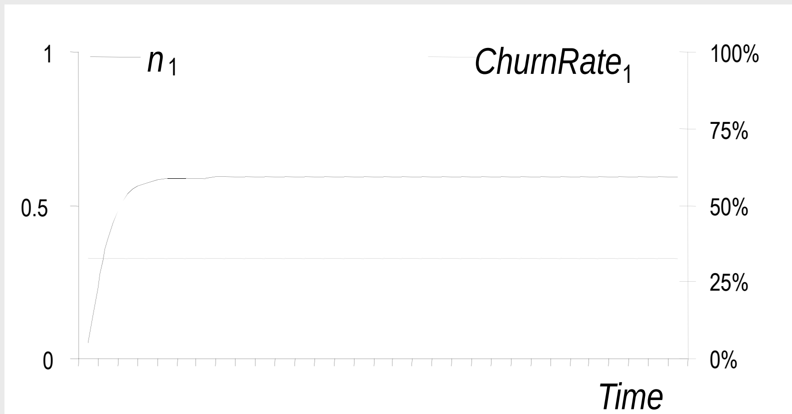
$$GrossAdd_i(t) = \sum_{j \neq i} N_j(t) \cdot p_{ji}, \quad i = 1,\dots,k; \; j = 0,\dots,k$$

$$NetAdd_i(t) = GrossAdd_i(t) - Churn_i(t), \quad i = 1,\dots,k$$

Example: Two operators



Churn rate for 1st operator



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Market Share Modeling and forecasting – MCDG Model

Markov chains based on diffusion growth (MCDG model):

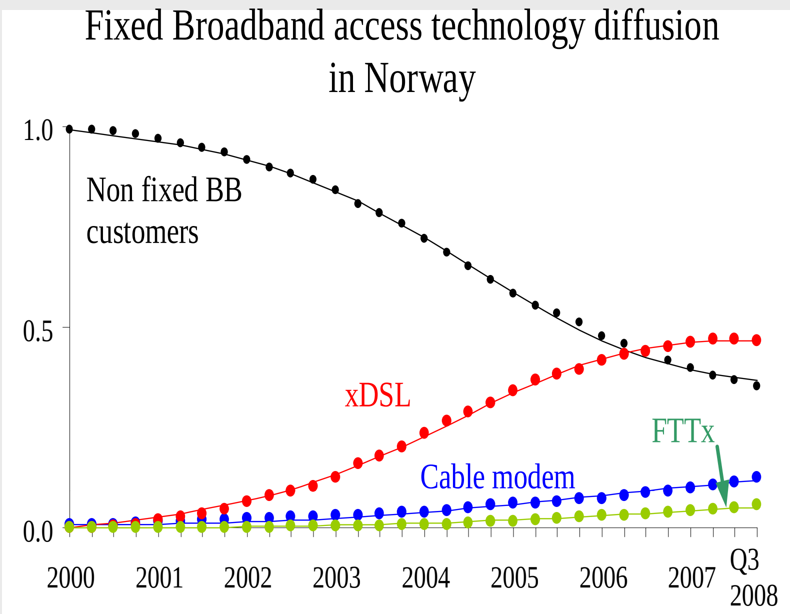
$$\left[n_0(t+\Delta t) \quad \cdots \quad n_k(t+\Delta t)\right] = \left[n_0(t) \quad \cdots \quad n_k(t)\right] \times \mathbf{P} + \left[n_0^2(t) \quad \cdots \quad n_k^2(t)\right] \times \mathbf{Q}$$

Matrices P and Q have the following elements:

$$\mathbf{P} = \begin{bmatrix} p_{00} & p_{01} & \cdots & p_{0k} \\ p_{10} & p_{11} & \cdots & p_{1k} \\ \vdots & \vdots & \ddots & \vdots \\ p_{k0} & p_{k1} & \cdots & p_{kk} \end{bmatrix}$$

$$\mathbf{Q} = \begin{bmatrix} 1-p_{00} & -p_{01} & \cdots & -p_{0k} \\ -p_{10} & 1-p_{11} & \cdots & -p_{1k} \\ \vdots & \vdots & \ddots & \vdots \\ -p_{k0} & -p_{k1} & \cdots & 1-p_{kk} \end{bmatrix}$$

Example:



Quality of modeling by MCDG measured via *RMSE* indicator is around 20 times higher than modeling by Markov chains!

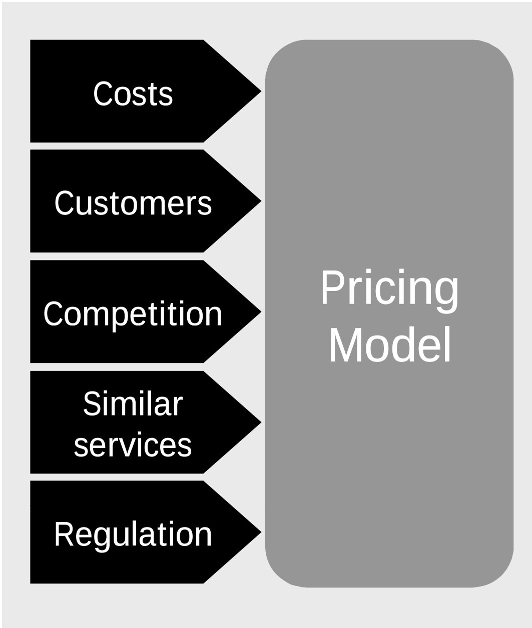
Pricing Models

- Principles
- *Fair-test*

Pricing models

Their main purpose is to adjust operator's offer to the market laws of demand. As a key for success in customer acquisition, retention and business in general, pricing model must encompass the following attributes:

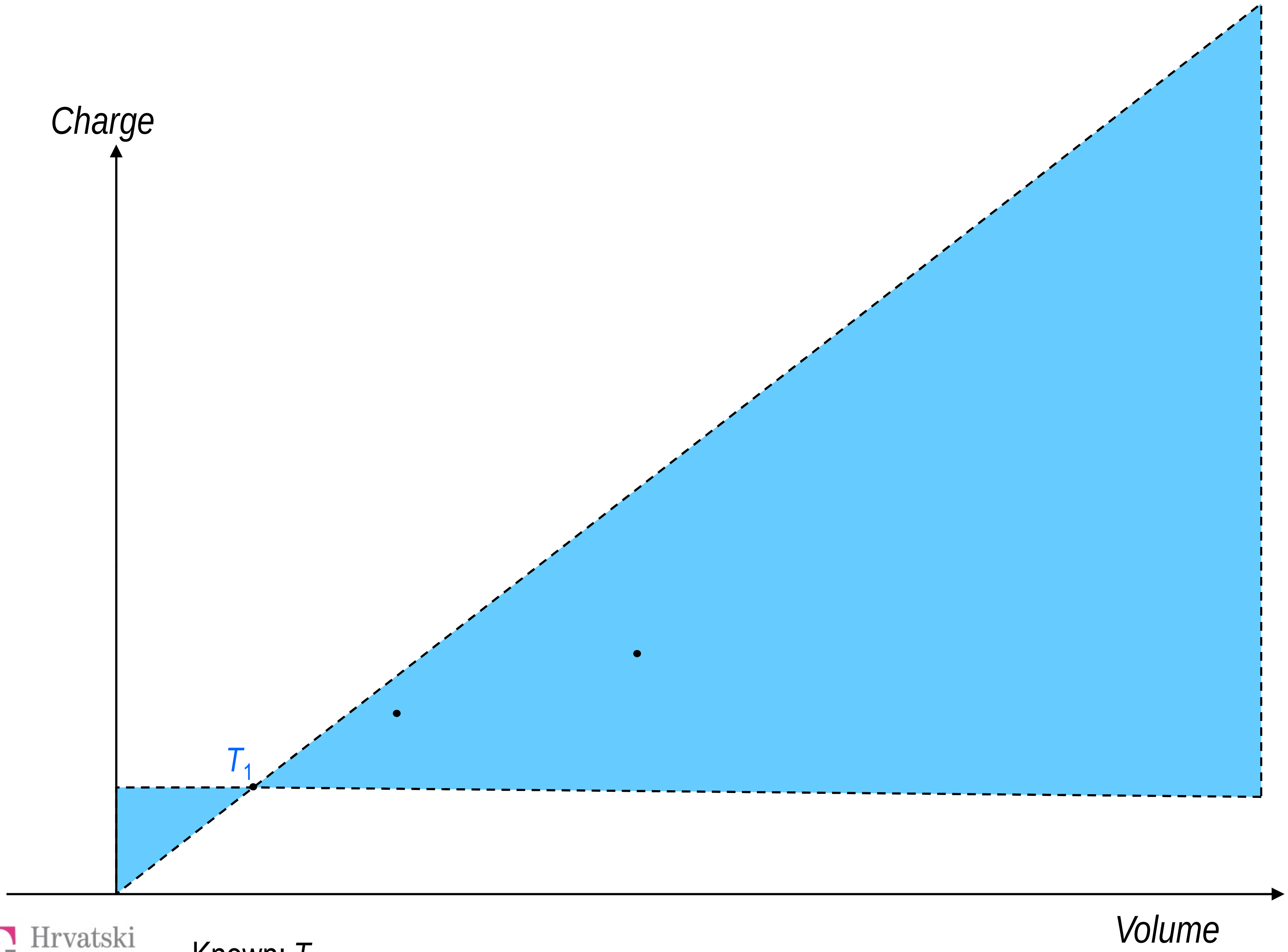
- Profitable,
- Billable,
- Flexible,
- Ensure large customer base,
- Easy to understand,
- Exploit willingness-to-pay,
- Consistent with regulation,
- Ensure competitiveness,
- Consistent with other services /pricing models in portfolio.



It must be fair in sense of usage:

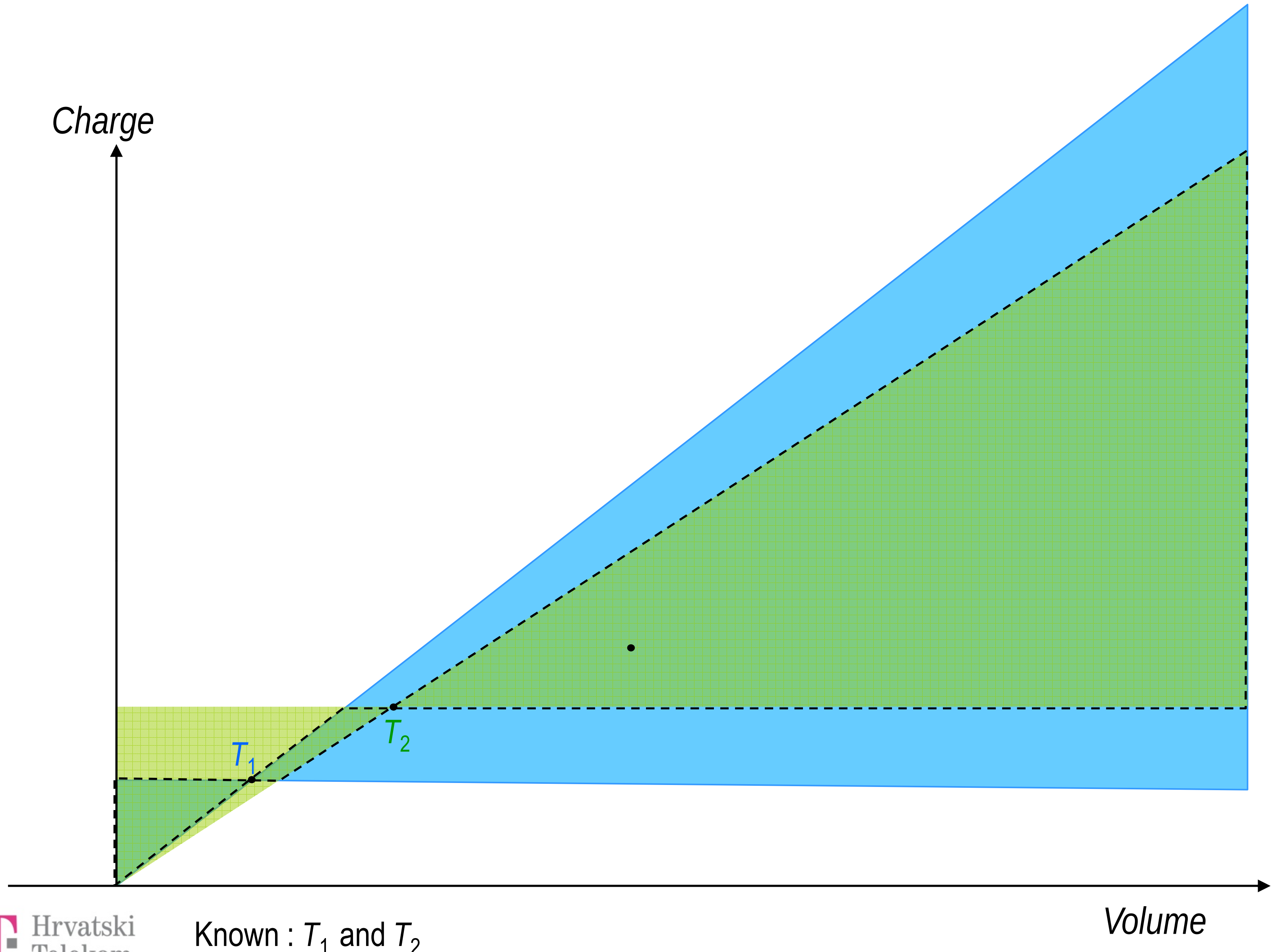
$$Charge(Volume_1 + Volume_2) \leq Charge(Volume_1) + Charge(Volume_2)$$

Pricing model Fair test



Known: T_1

Pricing model Fair test



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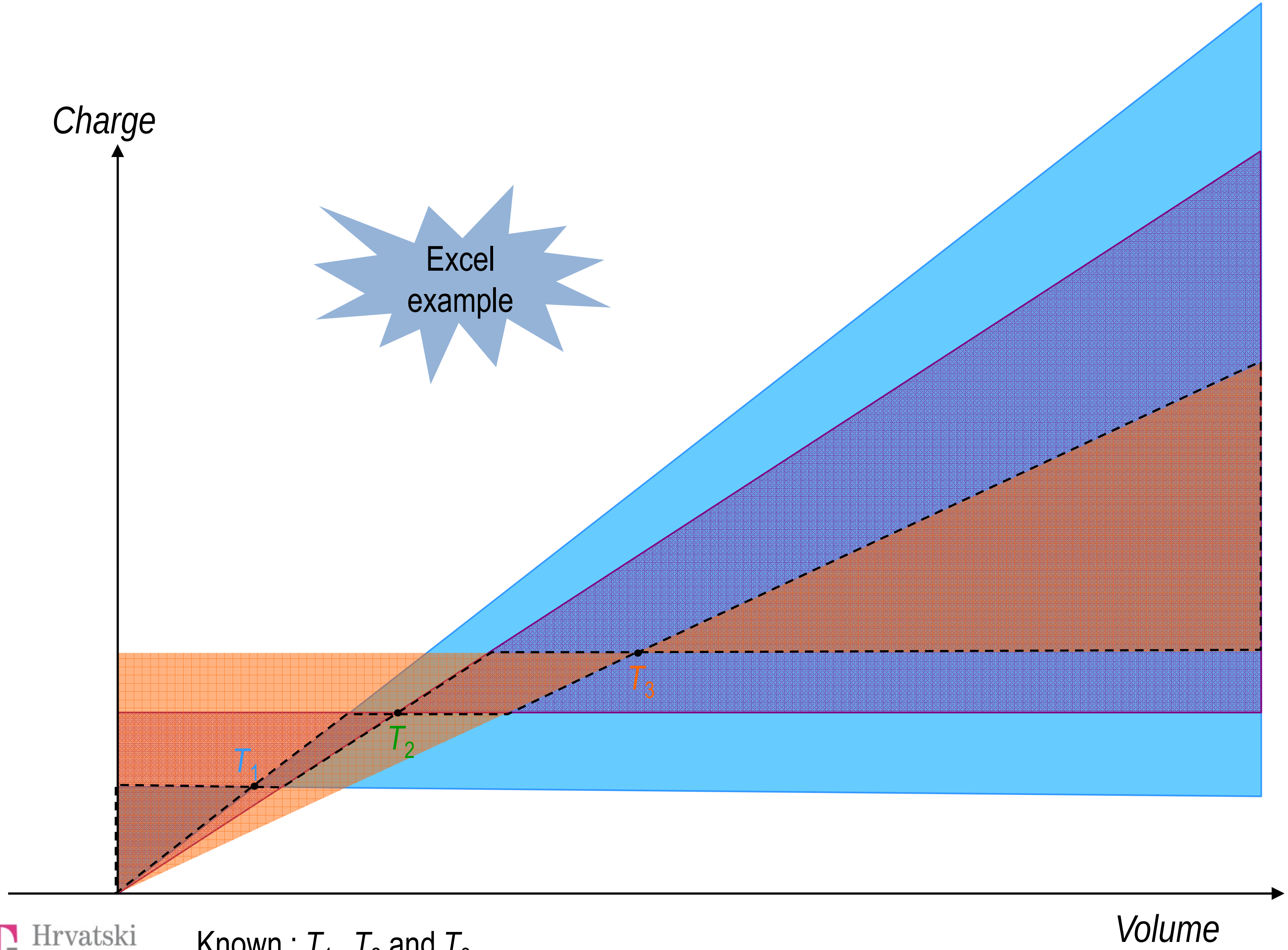
Known : T_1 and T_2

Pricing model Fair test



Known : T_1 , T_2 and T_3

Price elasticity of volume

$$E_V = \frac{\frac{dV}{V}}{\frac{dp}{p}}$$

p – unit price [*€/min, €/GB, €/SMS*]
V – realized volume of service [*min, GB, #SMS*]
R – revenue [€]

$$V = V_0 \left(\frac{p}{p_o} \right)^{E_V}$$

$$p = p_0 \left(\frac{V}{V_o} \right)^{\frac{1}{E_v}}$$

$$R = p \cdot V = R_0 \left(\frac{p}{p_0} \right)^{E_V + 1}$$

What should operator do to increase revenue?

For E_v = -0.5 - increase unit price

For E_v = -1.5 - decrease unit price